Università del Salento
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1. Research

In this period, according to the program under the project "Analisi algebrico-geometrica di modelli condensati multifase e con fase topologiche", I participated in investigations concerning three distinct problems, in collaboration with colleagues from the University of Salento (Professor Luigi Martina), from SISSA (Trieste, Professor Boris Dubrovin) and from Moscow State University (Professor Maxim Pavlov).

The main topics of the research were

i) the symmetrical approach to PDE's;

ii) integrable systems related to quantum models;

iii) Hamiltonian methods related to hydrodynamical systems.

The first task was the investigation of the Skyrme-Faddeev model (inspired by the infrared limit of the pure Yang-Mills particle model [1] and by a generalized 2-components order parameter Ginzburg-Landau model) by the method of the Lie group symmetry approach. Firstly the algorithm we create for computing the infinitesimal point transformations were tested on some simpler model, including the O(3) sigma-model. Its application to the Skyrme-Faddeev proved the absence of nontrivial point symmetries, except for the translations in space-time and rotations, both in spin and coordinate spaces. From these results we are performing the invariant symmetry reductions, in order to find special solutions. Moreover, special kind of solutions can be found also by suitable combinations of finite symmetry transformations, seen as special subgroups of the previous ones.

Then, we considered the problem of construction of nontrivial solutions by using the conditional symmetry approach, which was proposed by P. Olver and E. Vorobev [2], applied by L. Martina et al [3] in looking for integrability constraints. This method is based on the conjecture that infinitesimal transformations preserve solutions along characteristics. The main part of the algorithm providing the conditions of existence for such conditional symmetries was successfully derived. But still it remains to solve them, which we plan to perform in the nearest future. We expect to find out some nontrivial examples of solutions of topological structures in the Skyrme-Faddeev model.

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The second task was to find an integrability of the Skyrme-Faddeev system by "the method of hydrodynamic reductions".

The method of hydrodynamical reductions was proposed recently by E.Ferapontov, G.Gibbons, S. Tsarev et al. (in series of papers starting from [4]) for quasilinear multidimensional equations. The applications of this method to non integrable multicomponent quantum models was never done before.

The main idea of the method suppose a dependence of the model fields on a number of Riemann invariants only. The corresponding hydrodynamical system for the Riemann invariants involves unknown velocities (functions of the same invariants), which have to be determined by certain consistency conditions. The direct calculations provide beautiful geometrical restrictions on those velocities.

Restricting our considerations on the simplest nontrivial three Riemann invariants case, the corresponding solutions of the Skyrme-Faddeev system were found.

The third task was to find new classes of solutions for the WDVV associativity equation and linearly degenerate PDE's.

The WDVV associativity equation appears in the 2D topological field theory and in the Yang-Mills quantum model [5]. Futhermore, the solutions of the WDVV equation are connected with semi-Hamiltonian Egorov hydrodynamic type systems [5, 6]. We considered the problem of extracting the Egorov linearly degenerate hydrodynamic systems, equipped by local Hamiltonian structures associated with the Dubrovin-Novikov-Poisson brackets.

Such a class of systems was found and described by a family of constant parameters. In some simplest cases we found corresponding particular solutions of the WDVV associativity equations.

In a particular case of linear degeneracy, we have constructed a new type of solutions of the WDVV equation, containing infinite set of new rational solutions.

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2. Publications

1. G.A. El, A.M. Kamchatnov, M.V. Pavlov, S.A. Zykov: "Kinetic Equation for a Soliton Gas and Its Hydrodynamic Reductions", J. Nonlinear Sci. (2011) V.21: P.151–191.

2. B. Dubrovin, M. Pavlov, S. Zykov: "Linearly degenerate Hamiltonian PDEs and a new class of solutions to the WDVV associativity equations", submitted to the journal "Functional Analysis and its application" (May, 2011).

3. Participation to conferences and collaborations

NONLINEAR PHYSICS. THEORY AND EXPERIMENT. VI Gallipoli, June 23-July 3, 2010 (with the poster "Classification of integrable triangle hydrodynamic chains").

WASCOM 2011, Brindisi, June 12-18, 2011 (with a talk "Application of the hydrodynamic reduction method for the Skyrme-Faddeev system").

Visit to SISSA (Trieste), collaboration with B. Dubrovin and M. Pavlov, April 2011.

4. References

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systems, Comm. Math. Phys., v.248 (2004), p.187.
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[6] M.Pavlov, Integrable hydrodynamic chains, JMP (2003), v.44, p.4134.

Lecce, 28 giu 2011

Visto il tutor Luigi Martina